DC & RMS values of sinusoidal waveforms

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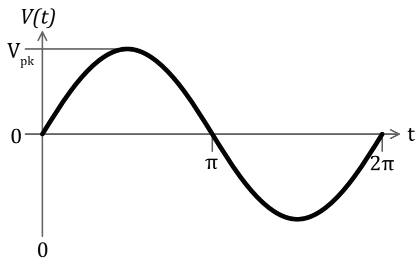
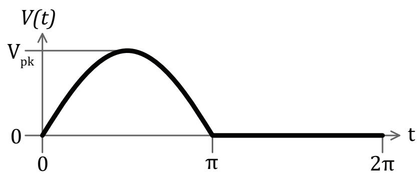
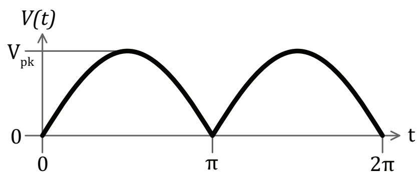
$\boldsymbol{V_{DC}}$ Average DC value of waveform

$\boldsymbol{V_{AC\ RMS}}$ RMS value of AC-only component

$\boldsymbol{V_{AC+DC\ RMS}}$ RMS value of AC+DC components (i.e. complete waveform)

Relation between V_{DC}, $V_{AC\ RMS}$, and $V_{AC+DC\ RMS}$:

$$V_{AC+DC\ RMS}^2 = V_{AC\ RMS}^2 + V_{DC}^2$$

Waveform	$\boldsymbol{V_{DC}}$	$\boldsymbol{V_{AC\ RMS}}$	$\boldsymbol{V_{AC+DC\ RMS}}$
	0	$\frac{1}{\sqrt{2}}V_{pk} \approx 0.707V_{pk}$	$\frac{1}{\sqrt{2}}V_{pk} \approx 0.707V_{pk}$
	$\frac{1}{\pi}V_{pk} \approx 0.318V_{pk}$	$V_{pk}\sqrt{\frac{1}{4}-\frac{1}{\pi^2}} \approx 0.385V_{pk}$	$\frac{1}{2}V_{pk} = 0.5V_{pk}$
	$\frac{2}{\pi}V_{pk} \approx 0.636V_{pk}$	$V_{pk}\sqrt{\frac{1}{2}-\frac{4}{\pi^2}} \approx 0.308V_{pk}$	$\frac{1}{\sqrt{2}}V_{pk} \approx 0.707V_{pk}$

Relation between V_{DC}, $V_{AC\ RMS}$, and $V_{AC+DC\ RMS}$

Any waveform can be represented by the sum of both an AC-only component, $V_{AC}(t)$, and a constant DC component, V_{DC}. The AC-only function could be obtained by high-pass filtering. The DC component is simply the time-averaged value of the waveform.

Many true RMS multimeters only measure the RMS value of the AC component of the waveform. This causes a large error when measuring a waveform which has a DC component, such as a half-wave rectified signal. To find the RMS value of the combined AC+DC components, use this expression:

$$V_{AC+DC\ RMS} = \sqrt{V_{AC\ RMS}^2 + V_{DC}^2}$$

$V_{AC\ RMS}$ is the value given by the meter and is the RMS value of the AC-only component. V_{DC} is the value given by the meter on the DC scale and is the time-averaged DC component.

Derivation:

Just to recap, remember that the RMS value of a periodic function is given by:

$$\sqrt{\frac{1}{T}\int_0^T [V(t)]^2 dt}$$

(Square the function, integrate it over one period (0 to T), divide by T to get the mean over a single period, then take the root of the whole lot.)

Represent our combined AC+DC function by:

$$V(t)_{AC+DC} = V(t)_{AC} + V_{DC}$$

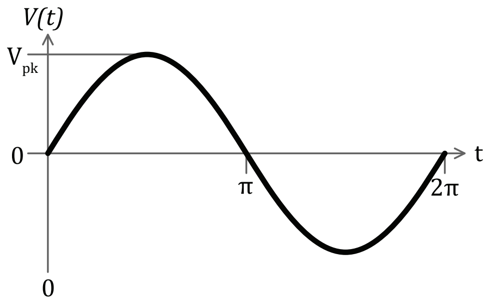
Proceed to calculate the RMS value:

$$\begin{aligned} V_{AC+DC\ RMS} &= \sqrt{\frac{1}{T}\int_0^T [V(t)_{AC} + V_{DC}]^2 dt} \\ &= \sqrt{\frac{1}{T}\int_0^T (V(t)_{AC}^2 + V_{DC}^2 + 2V_{DC}V(t)_{AC}) dt} \\ &= \sqrt{\frac{1}{T}\int_0^T V(t)_{AC}^2 dt + \frac{1}{T}V_{DC}^2\int_0^T dt + \frac{1}{T}2V_{DC}\int_0^T V(t)_{AC} dt} \end{aligned}$$

The first term is $V_{AC\ RMS}^2$, the second term is V_{DC}^2 and the last term is zero, because the average value of the AC-only component is zero. This gives the final expression:

$$V_{AC+DC\ RMS} = \sqrt{V_{AC\ RMS}^2 + V_{DC}^2}$$

Sinewave



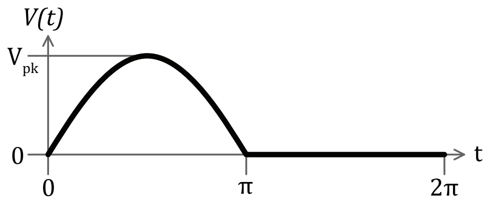
$V(t) = V_{pk} \sin t$

$$V_{DC} = \frac{1}{2\pi}\int_0^{2\pi} V(t)dt$$
$$= \frac{1}{2\pi}V_{pk}\int_0^{2\pi} \sin t\, dt$$
$$= -\frac{1}{2\pi}V_{pk}[\cos t]_0^{2\pi}$$
$$= -\frac{1}{2\pi}V_{pk}[1-1] = 0$$

$$V_{AC+DC\ RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V(t)^2 dt}$$
$$= V_{pk}\sqrt{\frac{1}{2\pi}\int_0^{2\pi} \sin^2 t\, dt}$$
$$= V_{pk}\sqrt{\frac{1}{2\pi}\frac{1}{2}\int_0^{2\pi} (1-\cos 2t)dt}$$
$$= V_{pk}\sqrt{\frac{1}{4\pi}\left[t-\frac{1}{2}\sin 2t\right]_0^{2\pi}}$$
$$= V_{pk}\sqrt{\frac{1}{4\pi}\left[2\pi-\frac{1}{2}\sin 4\pi - 0 + \frac{1}{2}\sin 0\right]}$$
$$= V_{pk}\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}V_{pk}$$

$V_{AC\ RMS} = V_{AC+DC\ RMS}$

Half-wave rectified



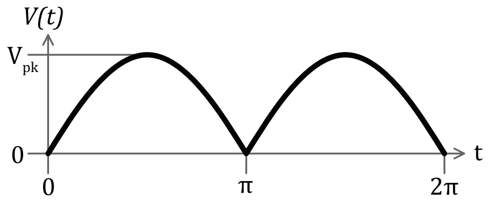
$$V(t) = \begin{cases} V_{pk}\sin t & 0 < t \le \pi \\ 0 & \pi < t \le 2\pi \end{cases}$$

$$\begin{aligned} V_{DC} &= \frac{1}{2\pi}\int_0^{2\pi} V(t)dt \\ &= \frac{1}{2\pi}\left[\int_0^{\pi} V_{pk}\sin t\, dt + \int_{\pi}^{2\pi} 0\, dt\right] \\ &= \frac{V_{pk}}{2\pi}\int_0^{\pi}\sin t\, dt \\ &= -\frac{V_{pk}}{2\pi}[\cos t]_0^{\pi} \\ &= -\frac{V_{pk}}{2\pi}[-1-1]_0^{\pi} = \frac{1}{\pi}V_{pk} \end{aligned}$$

$$\begin{aligned} V_{AC+DC\ RMS} &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V(t)^2 dt} \\ &= V_{pk}\sqrt{\frac{1}{2\pi}\int_0^{\pi}\sin^2 t\, dt} \\ &= V_{pk}\sqrt{\frac{1}{2\pi}\frac{1}{2}\int_0^{\pi}(1-\cos 2t)dt} \\ &= V_{pk}\sqrt{\frac{1}{4\pi}\left[t - \frac{1}{2}\sin 2t\right]_0^{\pi}} \\ &= V_{pk}\sqrt{\frac{1}{4\pi}\left[\pi - \frac{1}{2}\sin 2\pi - 0 + \frac{1}{2}\sin 0\right]} \\ &= V_{pk}\sqrt{\frac{1}{4}} = \frac{1}{2}V_{pk} \end{aligned}$$

$$\begin{aligned} V_{AC\ RMS} &= \sqrt{V_{AC+DC\ RMS}^2 - V_{DC}^2} \\ &= \sqrt{\frac{V_{pk}^2}{4} - \frac{V_{pk}^2}{\pi^2}} \\ &= V_{pk}\sqrt{\frac{1}{4} - \frac{1}{\pi^2}} \end{aligned}$$

Full-wave rectified



Only need to calculate things from 0 to π.

$$V(t) = V_{pk} \sin t$$

Average DC is twice that for the half-wave case:

$$V_{DC} = \frac{2}{\pi} V_{pk}$$

AC+DC rms is same as for full sinewave so:

$$V_{AC+DC\ RMS} = \frac{1}{\sqrt{2}} V_{pk}$$

$$\begin{aligned} V_{AC\ RMS} &= \sqrt{V_{AC+DC\ RMS}^2 - V_{DC}^2} \\ &= \sqrt{\frac{V_{pk}^2}{2} - \frac{4V_{pk}^2}{\pi^2}} \\ &= V_{pk} \sqrt{\frac{1}{2} - \frac{4}{\pi^2}} \end{aligned}$$