Force exerted by a band clamp

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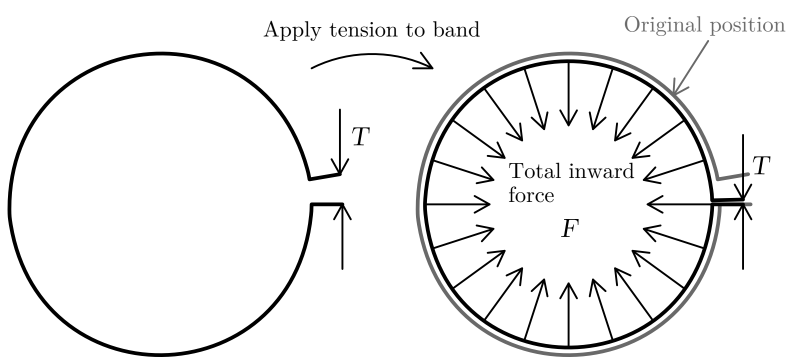
Introduction

A band clamp (*a.k.a.* hose clip/clamp or Jubilee clip) consists of a flexible band of metal wrapped around a cylindrical object (*e.g.* a hose, pipe, or circular container lid). As the band is tensioned (by whatever means is used by the specific design of clamp), the inner diameter decreases, exerting a large inwards force. Properly designed, band clamps distribute this force uniformly around the entire circumference.

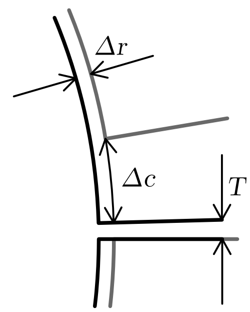
Problem

Given the tension in the band, T, what is the total inward radial force exerted, F?



Solution 1: Simple

The figure below shows a closeup of the end of the band as it is tightened. During tightening, the circumference decreases by an amount Δc and the radius by an amount Δr. Cicumference and radius are related by $c = 2\pi r$, therefore $\Delta c = 2\pi\Delta r$ or $\Delta r = \frac{\Delta c}{2\pi}$.

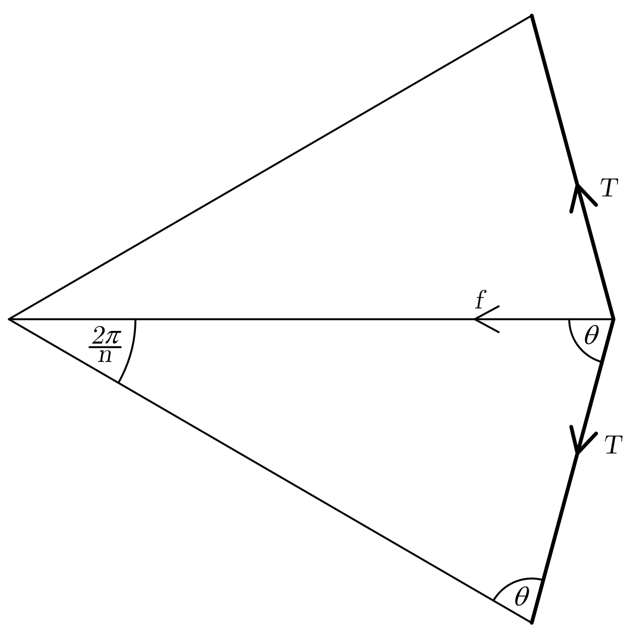


Since the radius changes by an amount that is smaller than the change in circumference by a factor of 2π, the inwards force, F, will be greater than the applied tension, T, by the same factor, so:

$$F = 2\pi T$$

Solution 2: Rigorous

Instead of wrapping the band around a cylindrical object, imagine it wrapped around an n-sided polygon. The figure below shows the forces acting on one corner of the polygon.



The force inwards on the corner, f, is given by

$$f = 2T\cos\theta$$

From the angles inside one of the triangles, we can determine θ to be

$$2\theta + \frac{2\pi}{n} = \pi$$

$$
\begin{aligned}
2\theta &= \pi - \frac{2\pi}{n} \\
\therefore \theta &= \pi\left(\frac{1}{2} - \frac{1}{n}\right)
\end{aligned}
$$

Substituting this into the expression for f, and noting that the total inwards radial force, F, is nf, we obtain

$$F = 2Tn\cos\left[\pi\left(\frac{1}{2} - \frac{1}{n}\right)\right]$$

In order to determine this for the case of a cylindrical object, we need to take the limit as $n \to \infty$ (since a circle is, effectively, a polygon with an infinite number of sides):

$$F = \lim_{n\to\infty} 2Tn\cos\left[\pi\left(\frac{1}{2} - \frac{1}{n}\right)\right]$$

Unfortunately, although $\lim_{n\to\infty}\cos\left[\pi\left(\frac{1}{2} - \frac{1}{n}\right)\right] = 0$, this is multiplied by n itself, giving the product $\infty * 0$. To solve this problem, we express the argument of the limit as

$$F = \lim_{n\to\infty}\frac{2T\cos\left[\pi\left(\frac{1}{2} - \frac{1}{n}\right)\right]}{\frac{1}{n}}$$

By using l'Hôpital's Rule (which states that the limit of a quotient with an indeterminate form is equal to the limit the quotient of the derivatives of the numerator and denominator), we differentiate top and bottom to obtain

$$F = \lim_{n\to\infty}\frac{-2T\frac{\pi}{n^2}\sin\left[\pi\left(\frac{1}{2} - \frac{1}{n}\right)\right]}{-\frac{1}{n^2}}$$

After simplifying, we obtain

$$
\begin{aligned}
F &= \lim_{n\to\infty} 2T\pi\sin\left[\pi\left(\frac{1}{2} - \frac{1}{n}\right)\right] \\
&= 2T\pi\sin\left[\frac{\pi}{2}\right] \\
&= 2\pi T
\end{aligned}
$$

which is the same result as obtained from the simple method.