

Bursting speed of rotating disc - relation to rotary spark gap

Dr. Lindsay Robert Wilson
<http://imajeenyus.com>

27/02/13

Stress in rotating disc

The best reference I could find for calculating the tensile stresses in a rotating disc is a set of lecture notes from the University of Tennessee at Martin. They are no longer on the web, but I got them through the Wayback machine at <http://web.archive.org/web/20040804122543/http://www.utm.edu/departments/engin/lemaster/Machine%20Design/Lecture%2016.pdf>. If that isn't available, see the file `lecture_16.pdf` in the same folder as this PDF on my website (link should work if viewed in a browser).

The lecture notes calculate the radial and tangential stresses in a solid rotating disc as

$$\begin{aligned}\sigma_r &= \frac{3 + \nu}{8} (b^2 - r^2) \rho \omega^2 \\ \sigma_\theta &= \frac{3 + \nu}{8} \left(b^2 - \frac{1 + 3\nu}{3 + \nu} r^2 \right) \rho \omega^2\end{aligned}$$

Units - if you use MKS units here, everything works out. Ignore the ridiculous imperial units used in the lecture notes.

σ is the tensile stress (Pa), ν is Poisson's ratio, b is the radius of the disc (m), r is the radius at which the stress is calculated (m), ρ is the density of the material (kg/m^3), and ω is the angular velocity (rad/sec).

We're interested in the maximum tensile stress, since that determines whether the disc bursts or not. Looking at the above equations, this occurs when $r = 0$, or at the center of the disc. Setting $r = 0$ and then swapping the symbol b with R (since it makes more sense to call the disc radius R), we get

$$\sigma_{max} = (R\omega)^2 \rho \left(\frac{3 + \nu}{8} \right)$$

To find the maximum speed a disc could rotate at, we set σ_{max} equal to the ultimate tensile strength and then solve for ω .

Application to rotary spark gaps

Here we assume the rotating disc has holes, not electrodes, and the holes are small enough that they don't affect the strength of the disc.

Consider a simple rotary spark gap consisting of two opposing fixed electrodes and a disc rotating between them with a number of holes. Let the number of holes on the disc be n , and the resulting firing frequency be f (which depends, obviously, on the number of holes and the rotational speed). In addition, let the separation/distance between adjacent holes on the disc be s . s has to be large enough to prevent a discharge occurring through a hole other than the one currently aligned with the fixed electrodes, and depends on the operating voltage.

The required circumference of the disc is ns , therefore the disc radius is

$$R = \frac{ns}{2\pi}$$

If we want a firing frequency of f and there are n holes on the disc, it will need to make f/n revolutions per second. Therefore, the angular velocity in radians per second is

$$\omega = 2\pi \frac{f}{n}$$

Substituting this back into the equation for the maximum tensile stress, we obtain

$$\begin{aligned}\sigma_{max} &= \left(\frac{ns}{2\pi} \times 2\pi \frac{f}{n} \right)^2 \rho \left(\frac{3+\nu}{8} \right) \\ &= (sf)^2 \rho \left(\frac{3+\nu}{8} \right)\end{aligned}$$

Rearranging to find sf , we obtain

$$sf = \sqrt{\frac{8\sigma_{max}}{\rho(3+\nu)}}$$

If you look at the RHS, it is only dependent on the properties of the disc material, so is a constant for a particular material. Pretty interesting! For a given material, the product of firing frequency and hole separation is a constant. It doesn't matter what you do to the size or rotational speed, the product sf is always a constant.

Examples

For polycarbonate, $\sigma_{max} \approx 70\text{MPa}$, $\rho = 1200\text{kg/m}^3$, $\nu = 0.37$. Therefore, $sf = 372\text{m*Hz}$ (units are meters*Hertz). If we needed a hole separation of 2cm, then the maximum firing frequency achievable with a polycarbonate disc is 18.6kHz. (Obviously, in practice, the safe limit would be much lower).

For FR4 glass/epoxy, $\sigma_{max} \approx 310\text{MPa}$, $\rho = 1850\text{kg/m}^3$, $\nu = 0.13$. Therefore, $sf = 650\text{m*Hz}$. A 2cm hole separation gives a maximum frequency of 32.7kHz.